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THESIS

A MODEL FOR DETERMINING OPTIMAL PREVENTIVE
MAINTENANCE INTERVALS FOR TANKS

by

Itzhak Raveh

December, 1979

Thesis Advisor: M. B. Kline

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The basis of the mathematical model developed is the assumption that a change in the frequency of treatment will cause a change in the number of failures. The model also takes into consideration other variables, such as mean time between maintenance (both preventive and corrective) and total number of treatments, subject to the constraints of availability and reliability. In addition, a method is presented for determining the optimum sample size for such an experiment.

The objective of the experiment, after collecting data and applying the mathematical model to the data, is to determine the optimal maintenance system--that system which yields the best ratio of combat effectiveness to cost.

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A Model for Determining Optimal Preventive Maintenance Intervals for Tanks

by

Itzhak Raveh
Lieutenant Colonel, Israeli Army
B.S., Technion-Israel Institute of Technology, 1973

Submitted in partial fulfillment of the
requirements for the degree of

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Author ______________________________

Approved by: Melvin B. Kline
Thesis Advisor

James D. Esary
Second Reader

EA Fincke
Chairman, Department of Administrative Sciences

D A Schrady
Dean of Information and Policy Sciences

ABSTRACT

A maintenance system is built on preventive and corrective maintenance. Of these two, preventive maintenance is that which permits planning in advance. It is based on scheduled treatments done at different levels and at different frequencies.

In tanks in action, preventive maintenance done at the intermediate level has the greatest influence on the efficiency of the maintenance system. In this study, the efficiency of the maintenance system will be checked by performing a controlled experiment, using different intervals between planned treatments at the intermediate level.

The basis of the mathematical model developed is the assumption that a change in the frequency of treatment will cause a change in the number of failures. The model also takes into consideration other variables, such as mean time between maintenance (both preventive and corrective) and total number of treatments, subject to the constraints of availability and reliability. In addition, a method is presented for determining the optimum sample size for such an experiment.

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List of Symbols

A	Availability
A_s	Availability of a single tank
a	Time measured in engine hours
c	Cost
d_G	Time to accomplish Treatment G
d_F	Time to accomplish Treatment F
d_λ	Time to accomplish corrective maintenance
$E(x)$	Expected value (mean)
$F(t)$	Probability of failure as a function of time
$F(a)$	Probability of failure as a function of engine hours
$f(t)$	Probability density function of occurrence of failure
i	Order number
MTBF	Mean time between failures
MTBM_{pt}	Mean time between preventive maintenance
MTTR	Mean time to repair (corrective maintenance)
$\overline{M}_{pt}$	Mean preventive maintenance time
m	Number of intervals in which Treatment F is performed
N_i	Number of tanks in the population
n_i	Number of tanks in the sample
$P(x)$	Probability function
r	Number of failures
R	Reliability
s	Specific time interval
T_γ	Time between consecutive Treatments G in engine hours
T_γ	Time between consecutive Treatments G in calendar months
$V(x)$	Variance
x	Random variable
Z	Z value for normal distribution
α	Confidence interval
γ	Average time between Treatments G
δ	Confidence level
Γ	$1/\gamma$, or frequency of occurrence of Treatment G
Θ	Average engine operating hours per day
λ	Failure rate
$\lambda(a)$	Failure rate as a function of engine hours
$\lambda(t)$	Failure rate as a function of time
λ'	Estimated failure rate from existing data
$\hat{\lambda}$	Maximum likelihood estimate of λ
Φ	Normal probability distribution
χ^2	Chi-square test statistic

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I. INTRODUCTION

The ability of an armored corps to accomplish its operational mission relies, in addition to its operational performance capability, on its ability to perform *when needed* (availability) and *for the duration of the assigned mission* (reliability).

Armed forces on the front lines, including tanks, are expected to be able to perform combat missions after a short warning time. For tanks, this means that a significant number of vehicles must be available, and each vehicle must be ready to carry out its assignments for a long period. This fact, and the fact that the modern tank is a complicated system which must operate under difficult environmental conditions, offer the maintainability designer a complex and challenging problem. The problems of availability, reliability, and maintainability are further complicated by the fact that not all tanks will be at the same level of reliability at any given moment.

In most modern armies under peacetime conditions, a certain percentage of the tank force will be in storage or in overhaul, or will be otherwise unavailable at the operating sites. Most of these tanks may be presumed to be at similar levels of reliability. The remaining tanks are occupied performing various operational and training missions, and, as a result, they do not all have the same level of reliability as those in storage. Each tank has its own level of reliability.

Taking into account the fact that these tanks have to join the order of battle as well as those tanks coming out of storage, the maintenance concept must provide for a way in which the tanks will stand up to

operational requirements even though they have different levels of reliability and availability. During a war, a similar but even more demanding condition prevails, since preventive and corrective maintenance must be done during the action and close to the front lines.

In the Israeli Armor Corps, which stays on the line of confrontation for long periods of time and which must maintain high availability levels, the determination of when to perform preventive maintenance, and how much effort to put into it, is of prime importance, especially when most of the tanks are operating daily and the cost of maintenance is high. Once the preventive maintenance policy is established, it is, for technical and organizational reasons, very difficult to change, despite the fact that there are many changes in operational requirements, maintenance costs, and the equipment itself. Therefore, it is important to reexamine from time to time whether the current policy meets operational demands at minimum cost.

The factors which should be examined by the developer of the maintenance policy can be divided into two groups:

(1) factors which are dictated to the designer, such as reliability and availability requirements, rate of training, cost of maintenance, and technical data;

(2) factors which the designer must consider based on past experience and development tests, such as service life, distribution of tank components, and the impact of preventive maintenance on the reliability of the tank.

A. OBJECTIVE

The objective of this thesis is to develop a mathematical model which will enable the evaluation of an existing maintenance policy for tanks, or which will assist in the development of a new, optimal policy. Although the model deals specifically with tanks, it can be applied to other systems to which a preventive maintenance and overhaul policy is applicable, such as airplanes, ships, and other ground vehicles.

B. BACKGROUND

In order to determine what information is of importance in designing for maintainability, it is necessary to delineate those factors which, when combined, make up maintenance tasks or actions. Maintenance activities may be partitioned into two major subsets: preventive maintenance and corrective maintenance.

Preventive maintenance is that maintenance performed, preferably on a scheduled basis, for the purpose of maintaining the tank in a satisfactory operating condition. It includes periodic tests, monitoring, servicing, and inspection. It is performed in the units, in intermediate workshops, and even in the depot, where overhauls are part of the preventive maintenance cycle.

Corrective maintenance is that maintenance performed to restore the tank to operating condition after a failure or other malfunction has occurred. Corrective maintenance includes fault detection, diagnosis, correction and verification. It is a critical area, because it involves the restoration of items which have failed to an operable state, often during a mission and within a relatively short time period.

PRIMARY SUBSETS OF MAINTENANCE ACTIVITIES

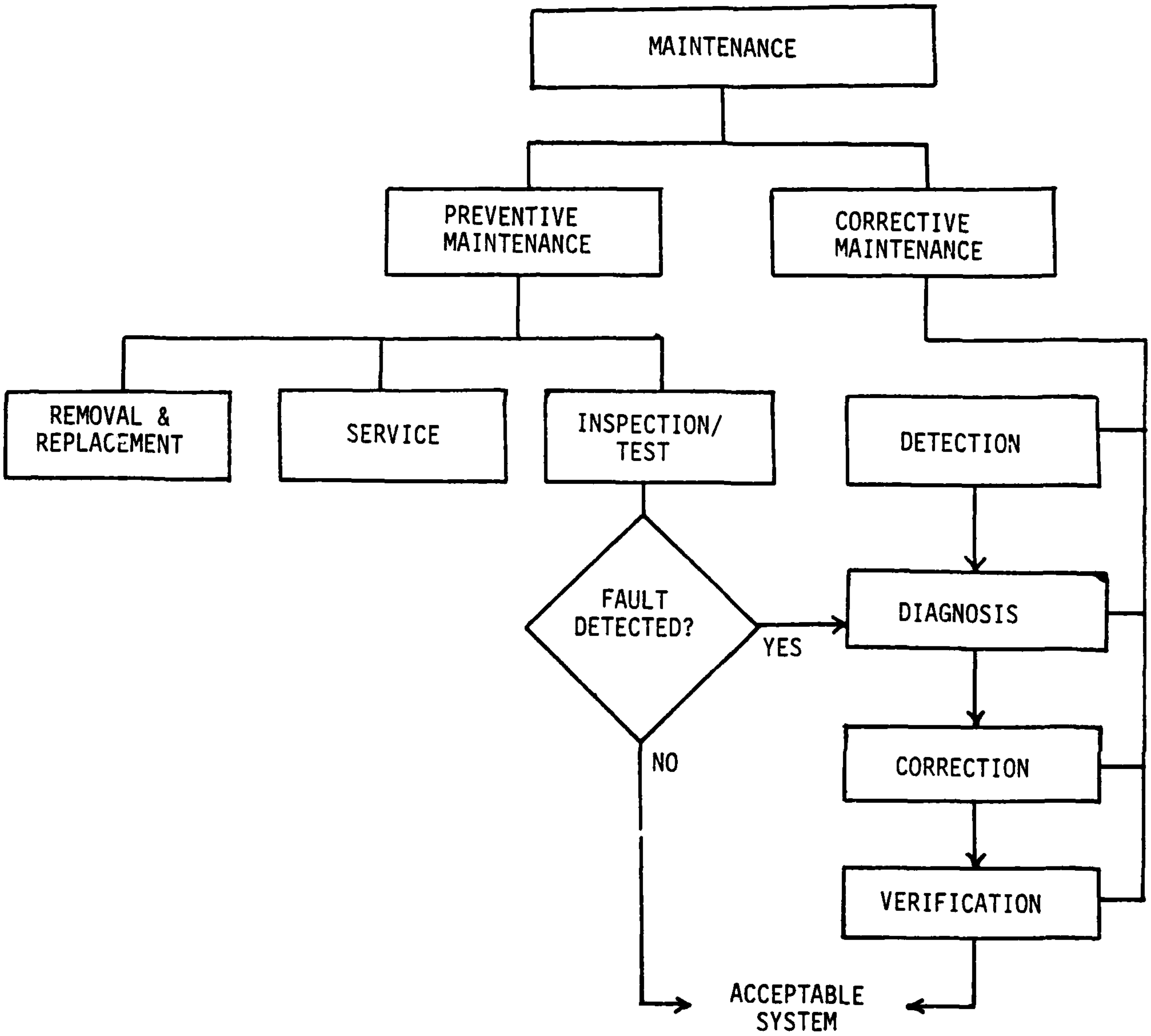

Figure 1

It is evident that time is the critical parameter in maintenance policy, and, therefore, an essential factor in maintainability design. Time enters maintainability considerations in two ways:

(1) in terms of long-term, or life, characteristics, and time between overhauls;

(2) in terms of short-term characteristics: the ability to keep an operating tank in operation (preventive maintenance), or to restore an inoperable tank to operational status (corrective maintenance).

Design for maintainability must include both preventive and corrective maintenance considerations. Because corrective maintenance is a function of preventive maintenance, and also because preventive maintenance is based on the concept of planning ahead, it is proper to commence the design of a maintenance policy with an evaluation of preventive maintenance.

In the Israeli army (as in the U.S. army), preventive maintenance functions are performed on three levels:

(1) Organizational

(2) Intermediate

(3) Depot

Organizational maintenance is performed at the operational site (in the field). Generally, it includes tasks performed by the using organization on its own equipment. Organizational-level personnel are usually involved with the operation of the tanks, and have minimum time available for detailed system maintenance. Maintenance at this level normally is limited to periodic checks of equipment performance, visual inspection, cleaning of equipment, external adjustment, removal and replacement of

some components, and some preventive servicing, such as changing oil and lubrication. Preventive maintenance carried out at this level is done with high frequency but takes only a short time.

Intermediate maintenance tasks are performed by mobile (in wartime) and fixed workshops. At this level the equipment is repaired by the removal and replacement of major modules, such as removing the engine, dismantling the recoil mechanism of the gun, and the like. Workshop maintenance is scheduled regularly for each tank, primarily after a tank has accumulated a certain number of engine hours or a certain number of months in the field. After such workshop preventive maintenance has been completed, the tank is returned to its operating unit and a new cycle is started. Maintenance tasks that cannot be performed at the lower levels, due to limited personnel skills and test equipment, are performed in the workshops. High personnel skills, additional test and support equipment, more spares, and better facilities often enable equipment repair down to the module and part level.

Depot maintenance constitutes the highest level of maintenance, and supports the accomplishment of tasks beyond the capabilities available at the intermediate level. The depot level of maintenance includes the complete overhaul, rebuilding, and calibration of the tank. The depot can also perform other highly complex maintenance functions. Depot maintenance is scheduled regularly for each tank, but with low frequency because it generally takes so much time. For the purposes of this thesis, the interval between two successive trips to the depot will be defined as the active operating cycle of the tank.

Today, in the Israeli army, the preventive maintenance accomplished at the intermediate level is divided into two main treatments, called Treatment F and Treatment G. The table below, although it does not represent a real situation, is approximately representative of the interval between preventive maintenance treatments.

Level of Preventive Maintenance	$MTBM_{pt}$ *Engine Hours*	$\overline{M}_{pt}$ *Days*	Cost *Working Days*
Organizational	5-10	1	30
Intermediate:			
Treatment F	200	7	500
Treatment G	200	14	900
Depot	600	40	2,000

Table 1: Maintenance Intervals

Since the purpose of the preventive maintenance treatment which takes place at the intermediate level is to assure continuity of the operational capability of the tank, and not necessarily to repair a failure (that being the function of corrective maintenance), the main factor which dictates the efficiency of preventive maintenance in the life cycle of a tank is the treatment itself, together with the interval between the performance of preventive maintenance services at the intermediate level. A long interval between G services will, on the one hand, reduce preventive maintenance cost and can increase the effective operational mean-time-between preventive maintenance ($MTBM_{pt}$), but, on the other hand, it might result in a decrease in reliability and availability and thus increase the need for corrective maintenance.

An example of a preventive maintenance time cycle between depot treatments (overhaul) is shown in Figure 2, where the letters "F" and "G" represent Treatments F and G, respectively.

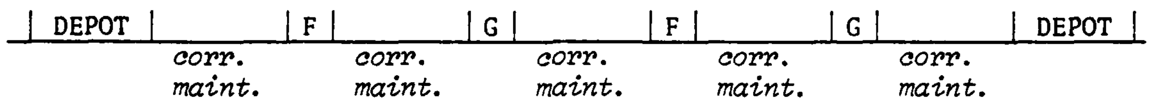

Figure 2: Preventive Maintenance Time Cycle

The main problem is to find the right trade-off between maintenance cost and the reliability/availability requirement, including the trade-off concerned with the decision as to whether a particular failure should be inhibited by means of preventive maintenance action or repaired by corrective maintenance action upon failure. Among other considerations, this trade-off is a function of failure modes, effects, and criticality, failure frequency, failure distribution, and mission and operational requirements.

C. THESIS APPROACH

Since the operational requirements are dictated by mission needs (from highest levels), the design of the maintenance policy will depend primarily on the impact that preventive maintenance has on operational requirements. Evaluation of the life cycle distribution of the tank components and the tank as a whole can be accomplished by performing an experiment.

In order to perform a controlled experiment and to evaluate the data, one should take the following steps:

(1) Define the operational policy objectives

(2) Define the main parameters that influence availability of the tank

(3) Design a test for estimating the parameters

(4) Perform the test and gather data

(5) Perform statistical analysis on the data

(6) Select feasible maintenance policies

(7) Determine the optimal policy

This study is concerned with the development of a model for defining the preventive maintenance problem. Such a model can help develop solutions to preventive maintenance policy problems. The study deals mainly with the first three steps enumerated above: defining the objective and designing the experiment. An appropriate design for an experiment to estimate the parameters which characterize the maintenance life length of a tank will involve a preliminary decision as to how to use the data gathered during the experiment. For this purpose, a basic model is constructed. After gathering the required data, on the basis of this model, the maintenance policy can be decided.

In order to construct the basic model, the maintenance policy of the Israeli army has been used. One may use any similar system to construct the basic model, or may use this basic model on each system which has a preventive maintenance cycle.

II. *THE BASIC MATHEMATICAL MODEL*

A. BASIC ASSUMPTIONS

N = Total number of tanks

N_0 = Tanks in operation that are actually qualified

N_1 = Tanks in operation, including corrective maintenance and Treatment F

N_2 = Tanks in Treatment G

N_3 = Tanks in depot

N_4 = Tanks in storage

$$\sum_{i=0}^{4} N_i = N \tag{1}$$

Notes: (1) We can, for a given maintenance policy, find the relationship among the various N_i's.

(2) The N_i tanks in status i can be divided into K sub-groups according to the different kinds of tanks and different kinds of operation.

In this study we will concentrate on only one kind of tank and one kind of operation. Furthermore, we will assume that each of the tanks of the N_i group will operate, on the average, Θ engine-hours per day.

B. MAINTENANCE POLICY

The maintenance concept on which this study is based is the "$T_\gamma;\mathcal{T}_\gamma$ concept," namely: Treatment G will be accomplished every T_γ engine-hours *or*

every T_γ months, whichever comes first. Treatment F will be performed at intervals between two Treatments G. If the number of Treatments F taking place between two consecutive Treatments G is m-1, where m is the number of intervals, then Treatment F is performed each (T_γ/m) engine hours or $(30T_\gamma/m)$ days.

The policy will be:

$$T_{days} = \min \left\{ \frac{T_\gamma}{\Theta} \; ; \; 30T_\gamma \right\} \tag{2}$$

Alternatively, if x_t is a random variable of the engine hours in the tth day of operation, then:

$$T_{days} = \min \left\{ \sum_{t=1}^{i} x_t \geqq T_\gamma \; ; \; 30T_\gamma \right\} \tag{3}$$

The expected value of the interval between two consecutive Treatments G for each tank will be $E(t)$, and the rate of Treatments G will be $\gamma = 1/E(t)$. A typical cycle with m-1 = 1 is given in Figure 3, where d_G is the time to accomplish Treatment G, d_F is the time to accomplish Treatment F, and d_λ is the time to accomplish corrective maintenance.

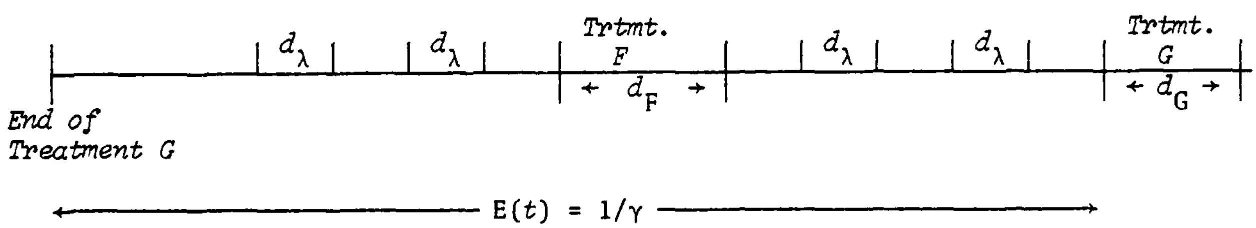

Figure 3: Typical Treatment Cycle

We may assume that the number of tanks receiving Treatment G at any given moment is given by:

$$N_2 = N_1 \gamma d_G \quad . \tag{4}$$

Also, we define λ as a failure rate, indicating the frequency with which malfunctions and failures occur, as follows:

$$\lambda = 1/\text{MTBF} \quad . \tag{5}$$

If a tank accomplished m-1 Treatments F between two consecutive Treatments G, then we may define $\lambda(a;t)$, as a failure rate of the tank between two consecutive Treatments F, as a function of both:

a = engine hours since the last Treatment G, and

t = time elapsed since the last Treatment G.

We assume that $\lambda(a;t)$ is a monotonic increasing function in both a and t; then, if T is the cycle length, $\lambda(\text{T})$ is the failure rate in the most "difficult" period for the tank, namely, just before its Treatment G.

We define the availability, A_s, of a single tank in a given moment $(a;t)$ when it is not in treatment as the proportion of the time that the tank is in a condition of fitness:

$$A_{(a;t)} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} = \frac{1/\lambda(a;t)}{1/\lambda(a;t) + \bar{d}_\lambda} \quad . \tag{6}$$

Also, because $\lambda(\text{T})$ is the worst maintenance condition that the tank is in, a basic requirement will be:

$$A_{(a;t)} \geq \frac{1/\lambda(\text{T})}{1/\lambda(\text{T}) + \bar{d}_\lambda} \quad . \tag{7}$$

For a given maintenance policy $(T_\gamma;\mathcal{T}_\gamma)$, the distribution of tanks according to time or engine hours will be given by the density function $f(a;t)$. Then, where $\overline{\lambda}$ is the average number of corrective maintenance actions in the given policy $(T_\gamma;\mathcal{T}_\gamma)$,

$$\overline{\lambda}_{(T)} = \iint_{ta} f(a;t)\lambda(a;t)da dt \quad . \tag{8}$$

To solve this equation and to find $\overline{\lambda}_{(T)}$, we will use the relationship between time and average engine hours per day: Θ. To illustrate the approach, we may assume that:

$$\lambda(a;t) = \lambda(a)(1 - e^{-\beta t}) \quad ,$$

and if

$$\lambda(a) = ca^2$$

and

$$f(a;t) = \frac{1}{T\Theta} \cdot \frac{1}{T} \quad ,$$

then:

$$\overline{\lambda}_{(T)} = \int_{t=0}^{T} \int_{a=0}^{T\Theta} \frac{1}{T\Theta} \cdot \frac{1}{T} \cdot ca^2(1 - e^{-\beta t})da dt$$

$$= \frac{cT\Theta^2}{3} \{T + \frac{1}{\beta}(e^{-\beta t} - 1)\} \quad .$$

By combining all of the above, we can find the proportion of time that the tank is in operation between two Treatments G:

$$A_g = 1 - \frac{(m-1)d_F + d_\lambda \overline{\lambda}_{(T)}/\gamma}{1/\gamma} = 1 - \gamma(m-1)d_F - d_\lambda \overline{\lambda}_{(T)} \quad .$$

A_s is the availability of a single tank. Therefore, at any given moment, the number of tanks actually qualified is:

$$N_0 = N_l A_s \quad . \tag{9}$$

C. OBJECTIVE FUNCTION

The objective function of the maintenance policy is to minimize the cost per unit time between two successive Treatments G, with regard to reliability and availability constraints (which will be defined in Section D, below). The cost per cycle can be expressed as an actual cost, or by accumulating the time for preventive and corrective maintenance (down-time), where:

c_G = cost of one Treatment G

c_F = cost of one Treatment F

c_λ = cost of one corrective maintenance

The cost of a cycle of length T+d will then be:

$$c_{T+d} = c_G + (m-1)c_F + c_\lambda \cdot \frac{\bar{\lambda}_{(T)}}{\gamma} \quad . \tag{10}$$

It is obvious that any change in maintenance policy (*i.e.*, an increase or decrease in $T_\gamma;\mathcal{T}_\gamma$ values) will cause a change in preventive maintenance cost. From this it follows that the objective function can be defined in one of the two following ways:

(1) Minimizing cost per unit time between two successive Treatments G, with average length of $(1/\gamma + d_G)$:

$$\min \left\{ N_1 \cdot \frac{c_G + (m-1)c_F + (c_\lambda \bar{\lambda}_{(T)}/\gamma)}{1/\gamma + d_\lambda} \right\} \tag{11}$$

(2) Minimizing the proportion of the average down-time in the interval between two Treatments G. In other words, maximizing the average availability in this interval:

$$\min \left\{ \frac{d_G + (m-1)d_F + d_\lambda \overline{\lambda}/\gamma}{1/\gamma + d_G} \right\} \qquad (12)$$

D. CONSTRAINTS

(1) The availability constraint for a single tank: The requirement is that, at any given moment, the availability of the tank will be equal to or greater than a given size, A_x. This availability is required for the worst case, namely, just before Treatment G. This is due to the assumption that some tanks will have to start the operation (war!) in this condition.

For a given T, the constraint will be as follows:

$$\frac{1/\lambda_{(T)}}{1/\lambda_{(T)} + d_\lambda} \geqq A_x \quad . \qquad (13)$$

This requirement is very severe; we may ease it by letting T_i become smaller than T:

$$\frac{1/\lambda_{(T_i)}}{1/\lambda_{(T_i)} + d_\lambda} \geqq A_x \quad , \quad (T_i<T) \quad . \qquad (14)$$

This means that for some tanks (those which passed T_i), the average availability will be lower than the required availability, A_x.

(2) If the constraint is defined as the availability not only at a given moment (*i.e.*, the outbreak of war) but after *y* hours from that moment, then we will define A′ as the required availability after *y* hours of operation:

$$\frac{1/\lambda(T+y)}{1/\lambda(T+y) + d_\lambda} \geqq A' \quad . \tag{15}$$

E. CONSTRAINT ON THE AVAILABILITY OF THE SYSTEM

Definition of the constraint: The requirement is that, for a given confidence level $1-\alpha$, from N_0 tanks which are qualified to operate, at least k tanks will still be qualified after x hours of operation (when $N_0 = N_1 A_s$).

We define $P_{N_0}(k,x)$ as the probability that, out of N_0 tanks, at least k will be in operation after x hours from the beginning of the assignment. If $F(t)$ is the distribution of the life length of the tanks, and if we assume that a hours have elapsed from the last Treatment G, then:

$$P(t \geqq x+a / t>a) = \frac{\overline{F}(a+x)}{\overline{F}(a)} \quad , \tag{16}$$

where $\overline{F}(a) = 1-F(a) = P(t \geqq a)$. Therefore the constraint will be:

$$R_{N_0}(k,x) = \sum_{j=k}^{N_0} \binom{N_0}{j} \left\{\frac{\overline{F}(a+x)}{\overline{F}(a)}\right\}^j \left\{1 - \frac{\overline{F}(a+x)}{\overline{F}(a)}\right\}^{N_0-j} \geqq 1 - \alpha \quad . \tag{17}$$

For example, if we assume that the life distribution is exponential with parameter $\overline{\lambda}_{(T)}$, then the constraint will be:

$$R_{N_0}(k,x) = \sum_{j=k}^{N_0} \binom{N_0}{j} \left\{e^{-\overline{\lambda}(T)x}\right\}^j \left\{1 - e^{-\overline{\lambda}(T)x}\right\}^{N_0-j} \geqq 1 - \alpha \quad . \tag{18}$$

Obviously, we can use other distributions in equation (17) to obtain the appropriate reliability.

III. *DESIGNING THE EXPERIMENT*

A. DETERMINING THE SAMPLE SIZE

To determine the sample size, we assume that the failure rate is exponentially distributed. (It should be emphasized that the model developed in Chapter II can be applied to any kind of distribution function. The analysis will be different, but the parameters and constraints will be applicable in any distribution.)

If the exponential distribution has parameter $\lambda(t)$, where t is measured in engine hours, then the probability that no failure will occur during an interval of time s is:

$$P = e^{-\int_0^s \lambda(t)dt} \quad . \tag{19}$$

The probability that k failures will occur during time s, according to the Poisson probability distribution, is:

$$P = \frac{1}{k!}\left\{\int_0^s \lambda(t)dt\right\}^k \cdot e^{-\int_0^s \lambda(t)dt} \quad , \tag{20}$$

and because of the assumption that the tanks are distributed homogeneously in the interval between zero and t, we define (with regard to engine hours):

$$\bar{\lambda}(t) = \frac{1}{t}\int_0^t \lambda(s)ds \quad , \tag{21}$$

or:

$$\overline{\lambda}(t)\cdot t = \int_0^t \lambda(s)ds \quad . \tag{22}$$

If $x(t)$ is a Poisson random variable with parameter $\lambda(t)\cdot t$, then we can, by combining equations (20) and (22), find the probability that j failures will occur in the time interval t:

$$P[x(t)=j] = \frac{e^{-\overline{\lambda}t}(\overline{\lambda}\cdot t)^j}{j!} \quad . \tag{23}$$

Based on the above, the required sample size will be:

$$P\left\{|\overline{x}(t)-\overline{\lambda}\cdot t| \leqq \alpha\cdot\overline{\lambda}t\right\} \geqq 1 - \delta \quad . \tag{24}$$

This requirement may be stated verbally as follows: The probability is greater than or equal to $1 - \delta$ that the parameter $\overline{\lambda}\cdot t$ is included in the interval $\{(\overline{x}(t)-\alpha\overline{\lambda}\cdot t);(\overline{x}(t)+\alpha\overline{\lambda}\cdot t)\}$. This interval is called the confidence interval, and the quantity $1 - \delta$ is the confidence level.

For the Poisson distribution, $V\{\overline{x}(t)\} = E\{\overline{x}(t)\} = t\cdot\overline{\lambda}$, the requirement will be:

$$P\left\{\frac{-\alpha t\cdot\overline{\lambda}}{\sqrt{t\overline{\lambda}/n}} \leqq \frac{\overline{x}(t)-t\overline{\lambda}}{\sqrt{t\overline{\lambda}/n}} \leqq \frac{\alpha t\overline{\lambda}}{\sqrt{t\overline{\lambda}/n}}\right\} \geqq 1 - \delta \quad . \tag{25}$$

In our case, when the sample size is large enough, we can use the normal approximation, and the requirement for the sample size will be:

$$\Phi\left(\frac{\alpha t\overline{\lambda}\sqrt{n}}{\sqrt{t\overline{\lambda}}}\right) \geqq 1 - \frac{\delta}{2} \quad \text{or} \quad n \geqq \left(\frac{Z_{1-\delta/2}}{\alpha}\right)^2 \cdot \frac{1}{\overline{\lambda}\cdot t} \qquad (26),(27)$$

We can see that the sample size, like the confidence level and the confidence interval, is a function of the experiment length t and the average failure rate $\bar{\lambda}$.

B. METHOD OF SELECTING THE SAMPLE SIZE

To determine the sample size (before the experiment), we need to estimate the failure rate λ' from existing data and to test the true failure rate $\bar{\lambda}(t)$ for a desirable confidence limit, γ. The experiment will consist of sampling with repairs. It will be performed with n tanks until time t, and during this period every failure that occurs will be corrected.

Let r_i be the number of failures occurring in a tank during t_i engine hours of an experiment. The maximum likelihood estimate for failure rate will be:

$$\hat{\lambda} = \frac{\Sigma r_i}{\Sigma t_i} \quad , \tag{28}$$

and if $\sum_i r_i = r$, then the two-sided confidence limits for a desirable $(1-\gamma)$ confidence level are given by:

$$\frac{2r\hat{\lambda}}{\chi^2_{1-\gamma/2}(2r+2)} \quad ; \quad \frac{2r\hat{\lambda}}{\chi^2_{\gamma/2}(2r)} \quad . \tag{29}$$

IV. *EXAMPLE EXPERIMENT*

In order to illustrate the use of the developed model, I will design a hypothetical experiment. As a basis for this experiment, four different alternative representations of the preventive maintenance cycle will be tested. By performing the experiment for each alternative, it will be possible to determine the failure rate $\lambda(a;t)$ as a function of time and engine hours from the last Treatment G. By using this failure rate and the mathematical model, it will be possible to decide which of the four alternatives represents the optimal maintenance policy.

The design of the experiment includes the following steps:

(1) Determining the maintenance concepts which will be tested

(2) Estimating the failure rate, taken from existing data (λ')

(3) Determining the specific sample size, and the duration of the experiment, for each alternative

(4) Determining the data to be collected

(5) Analyzing the data according to the model

(6) Determining the optimal alternative

A. MAINTENANCE ALTERNATIVES

In order to test different concepts in different ranges of engine hours and calendar time, the following four alternatives will be employed:

(1) Treatment G will be performed every 200 engine hours or 18 months. One Treatment F will be performed midway between each two consecutive Treatments G. See Figure 4.

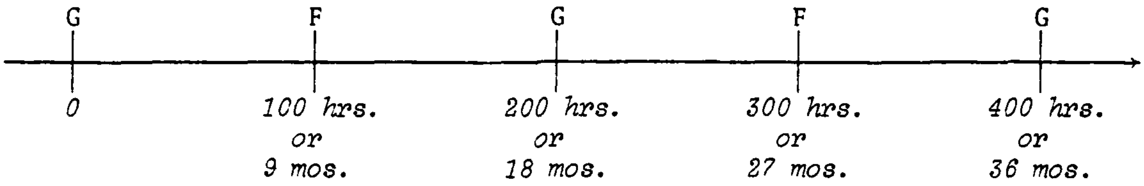

Figure 4: Diagram of Maintenance Alternative #1

(2) Treatment G will be performed every 400 engine hours, and Treatment F will be performed every 100 engine hours. See Figure 5.

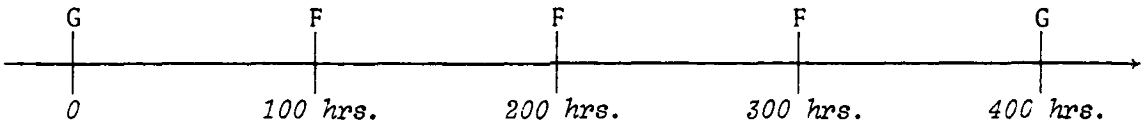

Figure 5: Diagram of Maintenance Alternative #2

(3) Treatment G will be performed every 200 engine hours or 18 months, whichever comes first. Treatment F will not be performed at all. See Figure 6.

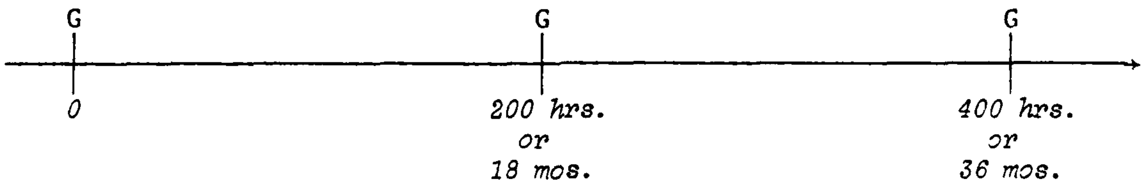

Figure 6: Diagram of Maintenance Alternative #3

(4) Treatment G will be performed every 300 engine hours or 27 months, whichever comes first. Treatment F will be performed every 150 engine hours. See Figure 7.

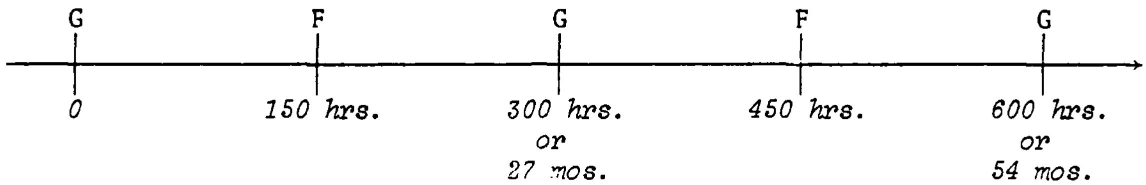

Figure 7: Diagram of Maintenance Alternative #4

In order to determine the failure rate under different maintenance conditions, we will use an existing maintenance concept and perform the experiment on operating tanks. For purposes of this illustration, we will assume that Alternative #1 is the existing maintenance concept.

The tanks taking part in the experiment should be divided into sub-groups according to their condition at the outset of the experiment and the length of the experiment:

(1) Sub-group n_1 will operate from one Treatment G to the next without an intervening Treatment F--total 200 hours or 18 months. See Figure 8. By performing this experiment we can determine $\lambda(a;t)$ in this interval both before and after Treatment G.

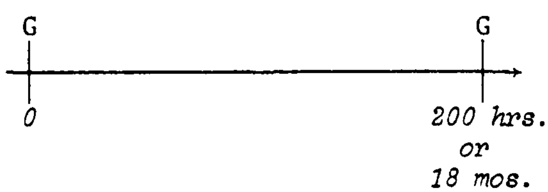

Figure 8: Sub-group n_1

(2) Sub-group n_2 will operate as sub-group n_1 with a Treatment F inserted midway between successive Treatments G. By performing this experiment we can determine $\lambda(a;t)$ with the influence of Treatment F between two consecutive Treatments G. See Figure 9.

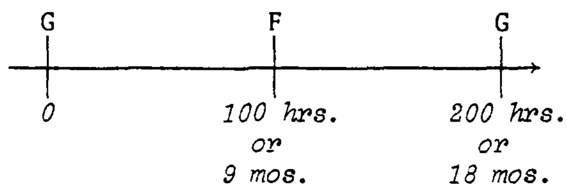

Figure 9: Sub-group n_2

(3) Sub-group n_3 will begin the experiment immediately following a Treatment F, which in turn was immediately preceded by another Treatment F. This group will operate for 300 engine hours while Treatments F and G are performed at 100-hour intervals. See Figure 10. By performing this experiment, we can determine $\lambda(a)$ from Treatment G until 400 engine hours.

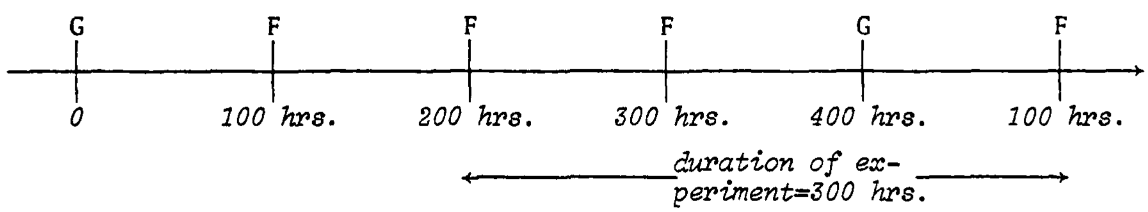

Figure 10: Sub-group n_3

(4) Sub-group n_4 will consist of tanks which have accumulated 100 engine hours since their last Treatment G. Treatment F will not be performed on these tanks, but they will be scheduled for Treatment G every 200 hours or 18 months, whichever comes first. The total duration of the experiment will be 300 hours or 27 months. See Figure 11. By means of this experiment we can determine $\lambda(a;t)$ using Treatment G only.

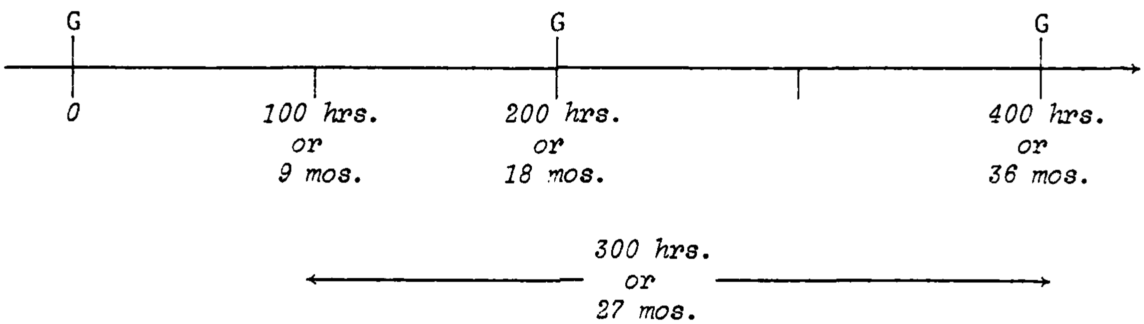

Figure 11: Sub-group n_4

(5) Sub-group n_5 will consist of tanks which have accumulated 100 engine hours since their last Treatment G. We will allow these vehicles to accumulate another 50 engine hours, and will then perform a Treatment F; then, after another 150 engine hours, we will perform another Treatment G. See Figure 12. This experiment will enable us to determine $\lambda(a)$ as a function of time within a 150-hour maintenance interval.

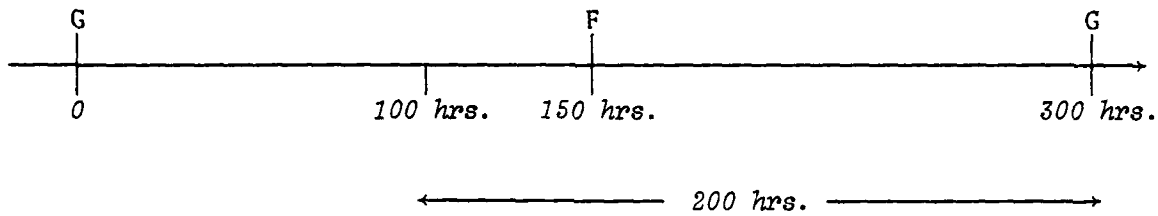

Figure 12: Sub-group n_5

B. ESTIMATING THE PARAMETERS WHICH INFLUENCE THE MAINTENANCE POLICY

To determine the sample size, we should estimate the failure rate from a similar population of tanks (λ'). In this example, the failure rate was calculated by observing the number of failures in a population of tanks similar to the population that took part in the experiment. These failures were categorized in two ways:

(1) According to the main systems of the tank in which they occurred --electrical, track/suspension, turret, and engine

(2) According to whether they took place before or after Treatment F

The failure data is presented in full in Appendix A. From it, we may derive the average failure rate, as follows:

Indicator for average failure rate = (total failures)/(total hours)

Total hours before Treatment F = 527

Total hours after Treatment F = 526

Total hours = 1,053

These failure rates are summarized in Table 2. Table 3 gives the value of the parameter $\left[\frac{Z_{1-\delta/2}}{\alpha}\right]^2$ for various values of α and δ, and Table 4 gives sample sizes as a function of λ' and experiment length.

FAILURES

	Total		*Electric*		*Track*		*Turret*		*Engine*	
	No.	*Rate*	*No.*	*Rate*	*No.*	*Rate*	*No.*	*Rate*	*No.*	*Rate*
TOTAL	36	.034	13	.012	10	.009	5	.0047	7	.007
BEFORE TREATMENT F	28	.053	12	.023	6	.0011	2	.004	7	.0013
AFTER TREATMENT F	8	.015	1	.002	4	.008	3	.006	0	--

Table 2: Failures by Categories

	0.01	0.05	0.1	0.15	0.2	0.25
$\delta = 0.1$	27,060	1,083	271	120	68	44
$\delta = 0.05$	38,416	1,537	384	169	96	62
$\delta = 0.01$	66,306	2,652	663	295	166	106

Table 3: Z-Parameter Values for Various Values of α *and*

$$\left[\frac{Z_{1-\delta/2}}{\alpha}\right]^2$$

	TOTAL $\lambda' = 0.034$				BEFORE TREATMENT F $\lambda' = 0.053$				AFTER TREATMENT F $\lambda' = 0.015$				ENGINE HOURS IN THE EXPERIMENT
	$\alpha=$ 0.05	$\alpha=$ 0.1	$\alpha=$ 0.15	$\alpha=$ 0.2	$\alpha=$ 0.05	$\alpha=$ 0.1	$\alpha=$ 0.15	$\alpha=$ 0.2	$\alpha=$ 0.05	$\alpha=$ 0.1	$\alpha=$ 0.15	$\alpha=$ 0.2	
$\delta = 0.1$	319	80	36	20	204	51	27	13	722	181	80	46	100
	159	40	18	10	102	26	14	7	361	91	40	23	200
	106	27	12	7	66	17	9	5	241	60	27	16	300
$\delta = 0.05$	452	113	50	28	290	73	32	18	1,024	256	113	64	100
	226	57	25	14	145	37	16	9	512	128	67	32	200
	151	38	17	9	97	24	11	6	341	86	38	21	300
$\delta = 0.01$	780	196	87	49	500	125	57	32	1,768	442	197	110	100
	390	98	44	25	250	63	29	16	884	221	98	55	200
	260	65	29	16	167	42	11	11	590	147	66	37	300

Table 4: Sample Sizes, $n = \left[\frac{Z_{1-\delta/2}}{\alpha}\right]^2 \cdot \frac{1}{t\lambda'}$

C. DETERMINING THE SAMPLE SIZE

In practice, determination of sample size is subject to the following considerations:

(1) The determination of the sample is based on the failure rate of the tank as a whole, and not on the failure rate of subsystems in the tank. It is possible to consider the failure rate of the specific subsystems, but this would yield a larger sample size requirement. See, for example, Appendix B, in which sample sizes are calculated taking into account only electric system failures.

(2) The number of tanks in the sample, for each of the alternatives, has been calculated in three ways:

  (a) for experiment length = 200 engine hours in all cases

  (b) for experiment length = 200 engine hours *or* 300 engine hours in those cases where the latter is desirable

  (c) for experiment length = 300 engine hours in all cases

(3) Values of α and δ were determined for reasonable sample sizes (*i.e.*, $\alpha = 0.1$ and 0.5, $\delta = 0.1$), but it is possible to calculate sample size for each desirable value of α and δ according to Tables 3 and 4.

Table 5 demonstrates several alternatives for selecting sample size.

	LENGTH OF EXPERIMENT					
	200 hours		200/300 hours		300 hours	
	α=0.1	α=0.15	α=0.1	α=0.15	α=0.1	α=0.15
n_1	20	9	N/A	N/A	N/A	N/A
n_2	20	9	N/A	N/A	N/A	N/A
n_3	40	18	27	12	27	12
n_4	40	18	27	12	27	12
n_5	40	18	27	12	27	12
total tanks in sample	160	72	121	54	109	48

Table 5: Sample Sizes by Sub-groups and Test Lengths

Note: $\delta = 0.1$ in all cases

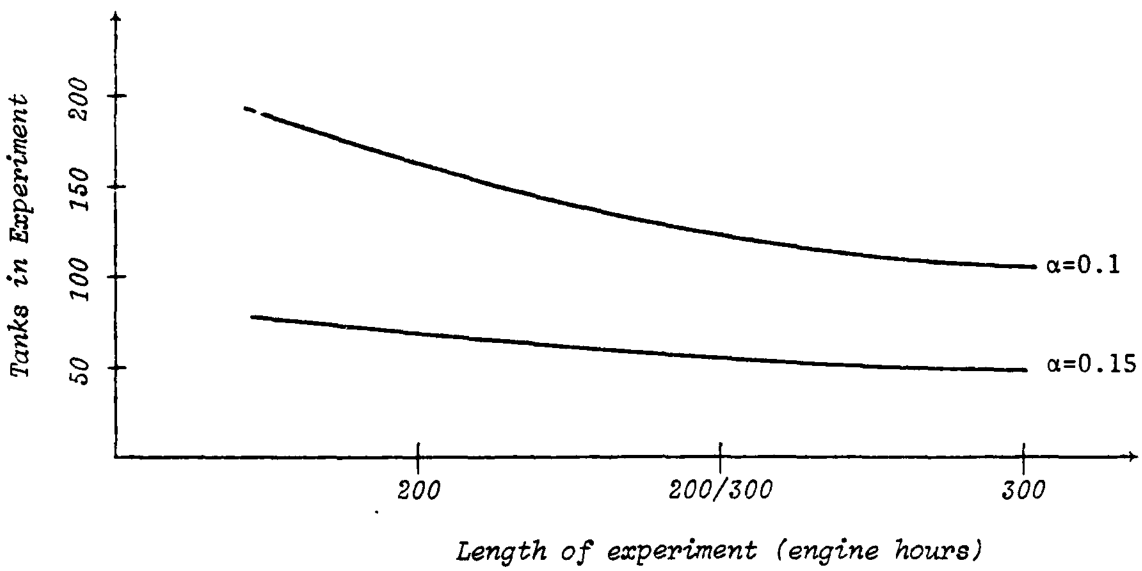

Figure 13: Graphic Depiction of Data in Table 5

From Figure 13, which graphically depicts the data given in Table 5, it is apparent that the determination of sample size in any particular case depends on the required confidence interval, α, and on the length of the experiment. An experiment of 300 hours with $100 \cdot (1-\alpha) = 85\%$ will require 48 tanks, while an experiment of 200 hours with $100 \cdot (1-\alpha) = 90\%$ will require 160 tanks.

D. DATA TO BE COLLECTED

After determining the sample size and the length of the experiment, we may begin collecting data. The principal items of data are:

(1) calendar time and engine hours from the beginning of the experiment until a failure appears, for each tank in the sample

(2) total time the tank is not in operation due to the failure, from the moment the failure appears to the moment the tank goes back into service

(3) net length of repair time and investment in parts and manpower

(4) length of time involved in Treatment F and Treatment G, and investment in parts and manpower

(5) engine hours on a day-by-day basis for each tank (Θ)

E. DATA ANALYSIS

From the collected data, it will be possible to draw the following conclusions:

(1) distribution of the failure rate function according to engine hours, $\lambda(a)$, and according to calendar time, $\lambda(t)$

(2) average engine hours per day

(3) common distribution function $\overline{\lambda}(a;t)$ from equation (8) for each of the different alternatives

(4) optimal maintenance policy from among the various alternatives presented

F. OPTIMAL MAINTENANCE POLICY

(1) According to the failure rate for each alternative, and reliability and availability constraints as dictated by operational requirements, it will be possible to check each of the alternative solutions to see whether it meets availability constraints for a single tank, using equation (13). The non-feasible alternatives will be discarded.

(2) The feasible alternatives should meet the reliability and availability constraints of the whole system, as expressed by equation (17), to a confidence level dictated by the operational demands.

(3) After categorizing the relevant alternatives, it will be possible to determine the optimal maintenance policy by using equation (12):

$$\min \left\{ \frac{d_G + (m-1)d_F + d_\lambda \overline{\lambda}/\gamma}{1/\gamma + d_G} \right\} .$$

Variables λ and m are fixed by the experiment for each alternative. Variables d_G, d_F, and d_λ are determined by the repair data for each alternative collected during the experiment, and $\overline{\lambda}$ can be calculated for each alternative.

The calculation of the optimal policy can be accomplished in the same way on a cost basis, using equation (11):

$$\min \left\{ \frac{c_G + (m-1)c_F + c_\lambda \bar{\lambda}/\gamma}{1/\gamma + d_G} \right\} .$$

V. *SUMMARY AND CONCLUSIONS*

An optimal maintenance policy for tanks is one which meets, at minimum cost, the operational requirements. Any such policy will, of necessity, be subdivided into preventive and corrective maintenance.

An operational tank, during the course of its life cycle, goes through a series of preventive maintenance services at different intervals and at different levels, and undergoes corrective maintenance whenever a failure occurs. In general, failures cannot be predicted in advance. In contrast, however, preventive maintenance can and should be planned in advance. Since failure rates are dependent, in large measure, upon the frequency and type of preventive maintenance, an optimal maintenance policy can be constructed on the basis of preventive maintenance schedules and policies.

Preventive maintenance operates on three main levels: organizational, intermediate, and depot. Depot treatment tends to be extensive, expensive, and infrequent; organizational maintenance, as a rule, is frequent but superficial. Therefore, as a basis for planning maintenance policy, we considered the intermediate level of maintenance. We estimated the principal parameters which influence maintenance policy, and used the interdependence among these parameters as the theoretical basis for the developed model.

The main parameter which dictates the nature of preventive maintenance policy is the interval between two Treatments G--the main preventive maintenance service which takes place at the intermediate level. This interval, in terms of engine hours or calendar time, largely determines the nature of the intervening Treatments F, and the distribution and nature

of corrective maintenance. It also has a great influence on availability, reliability, and cost. The basic assumption, in short, is that failure rate is determined as a function of the interval between preventive maintenance services.

Since failure distribution on the plane of time is a dependent variable and can be changed by any change in maintenance policy, it is necessary, in order to make a practical estimate of the failure rate $\overline{\lambda}$ under different maintenance policies, to conduct a controlled experiment. In such an experiment, in addition to the practical failure rate, we may observe other variables, such as corrective maintenance times, preventive maintenance times, and cost. The experiment should be "controlled" in the sense that actual, operational tanks are used, with no change in the nature of their activity, the only variable being the interval between their preventive maintenance treatments.

In order to perform such an experiment, and to obtain practical data from a minimum number of tanks and in a minimum amount of time, the model gives a method for selecting sample size as a function of experiment length and required level of accuracy. In principle, it is possible to choose, from among several alternative maintenance policies, that which is most cost-effective.

APPENDIX A

Details of Failures in 10 Tanks

Tank #	*Total Engine Hours*	*Total Failures*	*Treat- ment F*	*Engine Hours*	*Failures*	*Electric*	*Track/ Susp.*	*Turret*	*Engine*
1	116	7	before	61	6	2	3	0	1
			after	55	1	0	1	0	0
2	125	4	before	77	3	0	1	1	1
			after	48	1	0	1	0	0
3	124	2	before	62	1	1	0	0	0
			after	62	1	0	1	0	0
4	124	6	before	64	6	3	1	1	1
			after	60	0	0	0	0	0
5	125	5	before	60	4	4	0	0	0
			after	65	1	0	0	1	0
6	124	3	before	76	2	0	0	1	1
			after	48	1	0	0	1	0
7	45	1	before	34	1	0	0	0	1
			after	11	0	0	0	0	0
8	61	2	before	31	1	0	0	0	1
			after	30	1	0	0	1	0
9	95	4	before	32	3	2	0	0	1
			after	63	1	0	1	0	0
10	114	2	before	30	1	0	1	0	0
			after	84	1	1	0	0	0

APPENDIX B

Sample Sizes Considering Electric Failures Only

	TOTAL $\lambda' = 0.012$				BEFORE TREATMENT F $\lambda' = 0.023$				AFTER TREATMENT F $\lambda' = 0.002$				ENGINE HOURS IN THE EXPERIMENT
	$\alpha=$ 0.05	$\alpha=$ 0.1	$\alpha=$ 0.15	$\alpha=$ 0.2	$\alpha=$ 0.05	$\alpha=$ 0.1	$\alpha=$ 0.15	$\alpha=$ 0.2	$\alpha=$ 0.05	$\alpha=$ 0.1	$\alpha=$ 0.15	$\alpha=$ 0.2	
$\delta = 0.1$	902	226	100	57	471	118	52	30	5,415	1,355	600	370	100
	451	113	50	29	236	59	26	15	2,708	678	300	170	200
	301	76	43	19	157	39	18	10	1,805	452	200	111	300
$\delta = 0.05$	1,280	320	141	80	668	167	74	42					100
	640	160	71	40	334	89	37	21					200
	427	107	47	27	223	56	25	14					300
$\delta = 0.01$	2,210	552	246	138	1,153	288	128	72					100
	1,105	276	123	69	577	144	64	36					200
	740	187	82	46	385	96	43	24					300

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